Abstract: The development of an active sonar system requires knowledge of the properties of the waveforms transmitted by that system. The choice of waveform will determine the ability of the system to resolve targets in range and Doppler, and will also impact the detection capabilities of the system. Modulating the frequency or the amplitude of the transmitted waveform and coding the waveform with a pseudo random sequence will result in an increased time-bandwidth product. Amplitude modulation, however, results in a decrease of power efficiency, so it is more common to use frequency modulated and coded signals. Active sonar performance also depends also on reverberation rejection, and thus on pulse design. This article is based on Matlab simulations and primarily focuses on the resolution capabilities of signals coded using Farey sequences as well as their ability to reject reverberation and avoid mutual interference.

Key Words - Active Sonar, Matlab, Farey sequences, Costas coded, Matched Filter, Mutual interference Rejection.

I. INTRODUCTION

Active sonar creates a pulse of sound, called a ping and listens for reflections of the pulse to measure target parameters. The current situation is such that a single type of waveform cannot cater to requirements of accurate measurement of all the parameters required for analysing target properties, and as such the search for better class of waveforms assumes significance among researchers. In this paper, a deliberate attempt was made to review the performance of Costas coded waveforms for active sonar applications. The suitability of Farey sequence coding in active sonar applications was explored into, through a comparative study with the performance of Costas coded waveforms in active sonar applications. Farey Sequences are introduced in section II. Section III discusses the method of coding a waveform using Farey sequences, section IV describes the simulation set up used for analysis, and the methodology used to arrive at conclusions, section V compares Costas and Farey sequence(FS) coded waveforms of code length 10, based on their matched filter responses and signal quality characteristics, while sections VI and VII attempt to bring out a comparative study of Costas coded and Farey sequence coded waveforms based on reverberation levels and mutual interference rejection characteristics respectively. The paper concludes in section VIII with the expression of hope that Farey sequences will find potential use in active sonar systems in the days to come. Overall, this paper has attempted a meaningful discussion on the suitability of Farey sequence coded waveforms for potential use in active sonar systems in the near future.

II. INTRODUCING FAREY SEQUENCES

Farey sequences were named after the British geologist John Farey, whose letter about these sequences first appeared in the Philosophical Magazine in 1816. A Farey sequence of order $n$, $F_n = \{b_0, b_1, b_2, \ldots, b_{n}\}$ is a sequence of completely reduced fractions between 0 and 1 which, when in their lowest terms, have denominators less than or equal to $n$, arranged in the order of increasing size. Each Farey sequence starts with the value 0, denominated by the fraction 0/1, and ends with the value 1, denominated by the fraction 1/1. For example, the Farey sequences of orders 1 to 8 are:

$F_1 = \{0/1, 1/1\}$

$F_2 = \{0/1, 1/2, 1/1\}$

$F_3 = \{0/1, 1/3, 1/2, 2/3, 1/1\}$

$F_4 = \{0/1, 1/4, 1/3, 1/2, 2/3, 3/4, 1/1\}$
A very interesting property of Farey sequences is that if \( \frac{p}{q} \) and \( \frac{r}{s} \) are neighbours in the sequence with \( \frac{p}{q} < \frac{r}{s} \), their difference \( \frac{r}{s} - \frac{p}{q} = \frac{1}{qs} \).

### III. FAREY SEQUENCE CODING

Coding a waveform using Farey sequence (Figs 1a, 1b) implies varying or hopping instantaneous frequency of sinusoidal waveform in accordance with the sequence of selected order as per the expression:

\[
\hat{f}_i = f_b + B \times F_{s,i}
\]

where the base frequency \( f_b = 9800 + B b_0 = 9800 \text{ Hz} \) 

\[
(1 \ a)
\]

\[
f_i \text{ is the instantaneous frequency, and } B \text{ the bandwidth. The equation bears resemblance to that of a Costas}^{[1,4]} \text{ coded waveform. For simulations, a bandwidth of } 400 \text{ Hz with a centre frequency } f_i = 10 \text{ KHz of bandwidth and a pulse width of 0.5 seconds was assumed.}
\]

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Fig 1a: Farey Sequence Coded Waveform
The front end processor conditions the array signal and converts it to digital signals for processing. The receiver (Fig 2) has to process the signal at lesser data rates. The signal is bandpass filtered to improve SNR. Decimation process helps to sample the signal at a lesser data rate convenient to the receiver. The beamformer comprises a series of receiving elements or hydrophones. Receiving arrays are linear assemblies of hydrophone elements designed to increment signal to noise ratio and directionality. Array Weights and shading coefficients are multiplied element by element with the received signal to account for the attenuation suffered, to suppress side lobes and improve the directivity. The path difference issue between different receiving elements was accounted for at this stage. Chebyshev polynomial coefficients are used as shading coefficients to suppress side lobe levels to as low as 23dB below main lobe level and improve directivity.

From Transducer Array

**IV. ACTIVE SONAR RECEIVING CHAIN**

![Diagram](image_url)

*Fig 2 : Active Sonar Receiving Chain*
A normalisation algorithm\cite{5} was used to estimate and remove noise, and to make the noise background time invariant.

V. MATCHED FILTER RESPONSE

The correlation of doppler shifted stored reference with the received signal forms the matched filter response of a waveform. The matched filter responses and the observed parameters form the basis for the selection of a waveform suitable for active sonar detection. Range and Doppler resolutions are computed from the 3dB widths of the matched filter responses. Range resolution is given by the expression:

$$\Delta R = \frac{c \Delta \tau}{2}$$

(2)

where $c$ is the average sonic velocity in sea water and $\Delta \tau$ is the 3dB width of the matched filter response. Doppler resolution may be expressed as:

$$\Delta V = \frac{c}{(2f_c \Delta \tau)}$$

(3)

where $f_c$ is the centre frequency of the waveform. A waveform is said to be doppler sensitive if its matched filter response peaks only at the expected values of the delay and doppler parameter combination\cite{3}. Signal to Noise Ratio (SNR) and Signal to Reverberation Ratio (SRR) parameters have been derived from the peak values of matched filter responses normalised to rms mean values of background noise and reverberation signals respectively and expressed in decibels.

The detection capability of various ping types versus active signal spectrum reverberation may be modelled using Q-function, which is simply the integration of the ambiguity function along the range axis; i.e., the ability of the signal to discriminate against targets with different Doppler. Q-function is related to reverberation level by the expression:

$$RL = F(u_0)Q(\delta f_c)$$

(4)

$F$ is a function of range, $\delta$ the doppler parameter, $f_c$ the center frequency, and $Q$ is the $Q$-function at range $u_0$ and doppler shift $\delta f_c$. Hence a plot of reverberation levels become necessary to assess the performance of a waveform in a reverberation limited environment. A normalised logarithmic scale (base of 10) in decibels has been used for generating comparative merits of reverberation levels. Reverberation levels have been normalised to the total signal energy. The ratio of cross correlation of the stored replica to the interfering signal (or the matched filter response to the interfering signal) to the autocorrelation of the transmitted signal form the criteria for comparison of waveforms in terms of mutual interference rejection.

A. Signal Quality Measurements

The resolution capabilities for a given active waveform can be examined through the use of the ambiguity function\cite{1,3}. This function represents the envelope of the matched filter response to a target in both range and Doppler. The matched filter responses (Fig 3 and 4) have been normalised to the rms mean of background noise. The ability of a signal to resolve a target can be estimated based on the 3dB width of the main lobe of the ambiguity function. The matched filter response of a Farey Sequence (FS) coded waveform is shown in Fig 4. Sidelobe levels have been suppressed to 23 dB below main lobe level. Farey Sequence coded waveforms do exhibit Doppler sensitivity and their matched filter response (Fig 6) peaks only at the expected value of delay and doppler parameter(at $\delta = 0.02$) combination.
Approximate range (ΔR) and doppler (ΔV) resolutions of both Costas coded and Farey sequence coded waveforms have been estimated (Table 1) from the 3dB widths of the main lobe of the normalised matched filter responses.
### Table 1: Approximate Range ($\Delta R$) and Doppler ($\Delta V$) Resolutions

<table>
<thead>
<tr>
<th>Waveform</th>
<th>$\Delta R(m)$</th>
<th>$\Delta V(m/s)$</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Costas</td>
<td>3</td>
<td>19.5</td>
<td>Good resolution properties</td>
</tr>
<tr>
<td>FS Coded</td>
<td>3.375</td>
<td>16.666</td>
<td>Comparable to Costas, Slightly improved velocity resolution</td>
</tr>
</tbody>
</table>

Average Signal to Noise Ratio (SNR) and Signal to Reverberation Ratio (SRR) parameters in decibel scales have been estimated from the peak values of the respective matched filter responses and compared, as depicted in Table 2. The table highlights the superiority of Farey Sequence Coded waveforms over Costas coded waveforms in terms of signal quality characteristics.

### Table 2: Average SNR and SRR statistics.

<table>
<thead>
<tr>
<th>Waveform</th>
<th>Average Output SNR(dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Costas Coded</td>
<td>81.4731</td>
</tr>
<tr>
<td>FS Coded</td>
<td>77.2698</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Waveform</th>
<th>Average Output SRR(dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Costas Coded</td>
<td>105.0546</td>
</tr>
<tr>
<td>FS Coded</td>
<td>98.2282</td>
</tr>
</tbody>
</table>

### VI. REVERBERATION PERFORMANCE

Performance in reverberation limited environment of FS coded waveforms is very much comparable to that of Costas coded waveforms. A plot of normalised reverberation levels in decibels with doppler parameter delta varied between -0.5 and 0.5, as shown in Fig 5 confirms the fact that both Costas coded and FS coded waveforms exhibit similar degree of reverberation performance with varying doppler. Reverberation levels have been normalised to total signal energy.
VII. MUTUAL INTERFERENCE REJECTION

It is quite possible to have a scenario with multiple active sonar systems in the same area. It is always desirable to have an active transmit waveform that is not subject to high degrees of interference. The lower the interference, lesser the possibility of either jamming or target detection error. The matched filter response to different signal gives information about the mutual interference of different transmits waveforms. Hence, lower the value of the cross ambiguity function, lesser the interference and better the mutual interference rejection capability. Mutual interference rejection characteristics of Costas coded and Farey sequence coded waveforms against interfering frequency of 15 KHz are shown in Table 3.

Table 3: Cross Correlation Properties

<table>
<thead>
<tr>
<th>Reference Waveform</th>
<th>CW Correlation</th>
<th>LFM Correlation</th>
<th>STM Correlation</th>
<th>Costas Correlation</th>
<th>Cross Correlation</th>
<th>QCC Correlation</th>
<th>FS Coded Correlation</th>
<th>BFSK Correlation</th>
</tr>
</thead>
</table>

VIII. CONCLUSIONS

Statistics show that waveforms coded using Farey sequences tend to compete with Costas coded waveforms in many aspects. Waveforms coded using Farey sequences may be considered a potential candidate for use in sonar and radar target detection. Employability of Farey Sequence Coded waveforms in sonar applications will have to be investigated further based on probability of detection criteria. Again, more detailed analysis will be required before waveforms coded using Farey sequences can be strictly put into applications.
Fig 6: Doppler sensitivity characteristics of Costas coded Farey Sequence coded waveforms show doppler sensitivity.

X Axis: Doppler Parameter ($\delta$), Y: Peak of Matched Filter Response

REFERENCES


Mr. C K Sunith completed his M Tech Degree in Electronics from National Institute of Technology, Calicut. He has published one paper and has also contributed several articles to National and Regional Science and Technology magazines. He is a Life Member of Indian Society of Technical Education, an Associate Member of Institution of Engineers (India), Institution of Electronics and Communication Engineers (IETE) and a Life Member of Systems Society of India. Presently, he is working as Assistant Professor in Department of Electronics and Communication Engineering at Model Engineering College, Cochin.