Excitation Resonance Study of Laminated Composite Plates

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ABSTRACT—Structural elements subjected to dynamic excited in-plane loading may undergo unstable transverse vibrations for certain combinations of the values of the load parameters, i.e. the magnitude of the mean load, its amplitude and frequency of the pulsating component of the load. The dynamic in-plane loading is called the parametric excitation. The study of dynamic stability requires a special investigation of basic problem of vibration and static stability. The analysis of vibration regimes of a structure, subjected to in-plane periodic loading can be done through a simple analysis of dynamic instability region spectra.

Composite materials especially laminated composite plates have been widely used in various kinds of engineering such as aeronautic and marine structures and so on. The dynamic instability characteristics of stiffened plates subjected to in-plane uniform and concentrated edge loadings are studied using finite element analysis. In the structural modelling, the plate and the stiffeners are treated as separate elements where the compatibility between these two types of elements is maintained. The method of Hill’s infinite determinants is applied to determine the dynamic instability regions.

Numerical results are presented to study the effects of various parameters, such as static load factor, aspect ratio, boundary conditions, stiffening scheme and load parameters on the principal instability regions of stiffened plates using Bolotin’s method. The results show that location, size and number of stiffeners have a significant effect on the location of the boundaries of the principal instability region.

Keywords—Composite plate; vibration; dynamic stability; excitation; static load factor

I. INTRODUCTION

Composite materials are finding an ever-increasing application as critical structural members for aerospace structures, civil engineering structures. In addition to their high strength (or/and high stiffness) and lightweight, another advantage of laminated composite plate is the controllability of the structural properties through changing the fiber orientation and the number of piles or selecting proper composite materials, over a wide range. Structural elements subjected to dynamic in-plane loading may undergo unstable transverse vibrations for certain combinations of the values of the load parameters, i.e. the magnitude of the mean load, its amplitude and frequency of the pulsating component of the load. This type of dynamic instability is called parametric instability or parametric resonance. A review of laminated composite plates buckling has been done by Leissa [1987]. Buckling and optimization of layered composites has been studied by Iyenger [1989]. Recent advances in analysis of laminated beams and plates, and Shear effect and buckling has been studied by Kapania and Raciti [1987]. Recently, Reddy and Phan [1985] used a variational approach to derive a higher order theory in which a special displacement field is chosen to chosen stress free boundary conditions. A critical evaluation of new plate theories by Bert [1987], which provides an accurate prediction of non-linear bending stress distribution. Doong and Chen [1985] studied the effects of initial stresses on the vibration response of bi-modulus plates using Mindlin’s plate theory (first order shear deformation plate theory). The problems of buckling and free vibration of shear deformable unsymmetrical laminates was discussed by Noor [1975]. The dynamic instability of laminated composite plates was investigated by Cede Baum [1992] for uniform in-plane loading, using the method of multiple scales. The instability of laminated composite plates considering geometric non-linearity was also reported by Balamurugan and Ganapati [1996].

II. MATHEMATICAL FORMULATION

The elastic stiffness matrix \([K_p]\), geometric stiffness matrix \([K_{op}]\) and mass matrix \([M_p]\) of the plate element may be expressed as follows

\[
[K_p] = \int_{-1}^{+1} \int_{-1}^{+1} [B_p]^T [D_p] [B_p] J_p \, d\xi \, d\eta
\]
The equation of equilibrium for an elastic system undergoing small displacements at the instant of buckling may be written in matrix form as:

$$\left[ K_p \right] = \int \int \left[ B_{G_p} \right]^T \left[ \sigma_p \right] \left[ B_{G_p} \right] \left[ J_p \right] d\xi \, d\eta$$  \hspace{1cm} (2)

$$\left[ M_p \right] = \int \int \left[ N \right]^T \left[ m_p \right] \left[ N \right] \left[ J_p \right] d\xi \, d\eta$$  \hspace{1cm} (3)

The constitutive relations for the in-plane stress resultants and the stress couples are written in terms of the mid plane strains and curvature changes as: If \( \{ N \} \) represents the membrane stress resultants \( \{ N_{xx} , N_{yy} , N_{xy} \} \) where \( N_x \) and \( N_y \) are in-plane normal forces (resultants), and \( N_{xy} \) is the shear stress; \( M_x \) and \( M_y \) are in-plane bending moments resultants, and \( M_{xy} \) is the twisting moments, one can relate these to the membrane strain \( \{ \varepsilon_p \} \) and bending strain \( \{ \varepsilon_b \} \) through the constitutive relation as:

$$\{ N \} = \left[ A_{ij} \right] \{ \varepsilon_p \} + \left[ B_{ij} \right] \{ \varepsilon_b \}$$  \hspace{1cm} (6)

$$\{ M \} = \left[ B_{ij} \right] \{ \varepsilon_p \} + \left[ D_{ij} \right] \{ \varepsilon_b \}$$  \hspace{1cm} (7)

### III. RESULTS AND DISCUSSION

Rectangular Composite plates under general uniformly distributed in-plane edge loading are considered as shown in figure 1 to study the vibration and dynamic stability behaviour of composite plates. The loading shown in figure 1 is compressive in nature.

![Figure 1 Uniform edge compression.](image_url)

In a finite element analysis, it is desired to have the validation studies to estimate the accuracy of the numerical solution. In the results, buckling load is expressed in the form of non-dimensional buckling load parameter. The validation study has been carried out for buckling loads parameter for a composite plate subjected to uni-axial in-plane loading for various plate geometries and load positions as shown in table 1, and are compared with the results of Phan and Reddy [1985], and Srivastava and Krishnamurthy [1992].

Table 1 shows values of buckling load parameter \( \lambda \) for \( [0^\circ / \pm 45^\circ / 90^\circ] \), square laminates for various boundary conditions. The variation of \( \lambda \) is found to depend on stacking sequence. It has been observed that in case of \( [0^\circ / 90^\circ] / 0^\circ \) laminates the variation of \( \lambda \) with number of layers is almost negligible.

A perusal of these results indicates that a mesh size of 10x10 is sufficient enough to get a reasonable order of accuracy. The analysis in the subsequent problems is carried out with this mesh size.

<table>
<thead>
<tr>
<th>Boundary Conditions</th>
<th>Buckling load Parameter (( \lambda ))</th>
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<th>Buckling load Parameter (( \lambda ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-S-S-S</td>
<td>43.16</td>
<td>S-C-C-C</td>
<td>71.46</td>
</tr>
<tr>
<td>C-C-C-C</td>
<td>133.54</td>
<td>C-C-F-C</td>
<td>41.74</td>
</tr>
<tr>
<td>C-C-S-C</td>
<td>90.45</td>
<td>S-F-S-F</td>
<td>21.75</td>
</tr>
<tr>
<td>C-S-C-S</td>
<td>114.73</td>
<td>C-F-C-F</td>
<td>89.41</td>
</tr>
</tbody>
</table>

Table 1. Buckling load parameter \( \lambda \) for square laminated plates \([0^\circ / \pm 45^\circ / 90^\circ]\)
The effect of different parameters on dynamic instability region of composite plates is studied in this section. It includes the static and dynamic load factors, plate aspect ratio, boundary conditions and number of layers, stacking sequence. Results are presented in graphical form where the instability region is shown by the upper and lower values of the non-dimensional excitation frequency parameter \( \Omega = \left( \frac{\alpha a^2}{h} \right) \left( \sqrt{\frac{\beta}{E_2}} \right) \) where \( E_2 \) is the young’s modulus normal to the fibre direction of the orthotropic material and \( h \) is the thickness of the laminates. In all these cases, the frequency parameter is plotted against dynamic load factor (\( \beta \)) for different values of the parameters. The dynamic instability behaviour of cross-ply and angle ply laminated composite plate under uni-axial harmonic in-plane loads are investigated using finite element analysis. For the validation of dynamic instability, a simply supported square plate is analyzed with different static and dynamic load factors. The boundary frequencies obtained in the present analysis are presented with those of Hutt and Salam [1971] in table 2, which show good agreement.

### Table 2: Comparison of principal instability regions of a simply supported square plate subjected to uniform in-plane edge loading (uni-axial) for different static load factors (\( \alpha \)).

<table>
<thead>
<tr>
<th>( \alpha = 0 )</th>
<th>( \alpha = 0.6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>( \beta )</td>
</tr>
<tr>
<td>U</td>
<td>L</td>
</tr>
<tr>
<td>0</td>
<td>39.46</td>
</tr>
<tr>
<td>0.4</td>
<td>43.16</td>
</tr>
<tr>
<td>0.8</td>
<td>46.54</td>
</tr>
<tr>
<td>1.2</td>
<td>49.54</td>
</tr>
</tbody>
</table>

The dynamic instability studies are done for isotropic, quasi-isotropic, cross-ply and angle-ply plates of a moderately thick size (b/h = 25) subjected to uniform un-axial loading applied at the plate boundary throughout the investigation. A standard case is defined for the problem in which the geometrical and material properties are as follows: \( E_1/E_2 = 40.0 \), \( G_{23} = 0.5 E_2 \), \( G_{12} = G_{13} = 0.6 E_2 \), \( V_{12} = 0.25 \), \( a = 250 \) mm, \( b = 250 \) mm, \( h = 10 \) mm.

The instability results of cross-ply plates with a ply lay up of \( (0°/90°/90°/0°) \) are taken as the standard case and compared with the literature. Then the study is extended to the instability criteria of a quasi-isotropic material with a ply lay up of \( 0°/-45°/45°/90° \).

The parametric instability is plotted for a uniformly loaded antisymmetric angle-ply plates and cross-ply plates to consider the effect of the number of layers.

The effect of the static component of load (\( \alpha = 0.2, 0.4, 0.5, 0.6 \)) on the instability region is shown in figure 3. It is observed that with the increase of the static load factor from 0.2 to 0.6, the onset of dynamic instability occurs earlier and the width of the dynamic instability region also increases. It is observed that due to an increase in the static component, instability regions tend to shift to lower frequencies and become wider. The same effects are observed for any number of stacking sequence, boundary conditions, aspect ratios, considering or neglecting the in-plane displacements. It is also observed that the boundary frequencies values for the laminated composite plate considering in-plane displacement are less than for neglecting the in plane displacements corresponding to any value of static load factor. All further studies were made with a static load factor of 0.2 (unless otherwise mentioned).

The effect of boundary conditions on the instability region is shown in figure 4. It is observed that the onset of instability occurs later with narrow zones of instability and with the addition of restraint at the edges. So for the same aspect ratio, the excitation frequency of composite plate with all edges clamped will be more than those other boundary conditions. It can also be observed that the more rigid boundary conditions have greater stability.

The effect of the number of the layers is shown in figures 5 for clamped composite plate. The lamination scheme is \( 45°/45° \) for the two-layer plate and \( 45°/45°/45°/45° \) for the four-layer plate for \( a/h = 25 \). It observed from figure 5 that the structure is more stable under periodic loads with an increase in the number of layers, showing instability regions at the higher frequencies and narrower, which compares well with the literature. The instability region has lower frequencies and is wider for the two-layer than four-layer case. The increase of the number of layers shifts the instability region to larger frequencies. The increase in the number of layers reduces the instability regions.
The observed behaviour is attributed to the effects of bending stretching coupling for the case of laminates, due to a decrease in effective stiffness.

**Figure 3**  Effect of static load factor on dynamic stability region for laminated composite plate subjected to uniform edge loading.

**Figure 4**  Effect of boundary condition on dynamic stability region for composite square plate $\alpha = 0.2$, subjected to uniform edge loading.

**Figure 5**  Effect of number of layers on dynamic instability region of a simply supported laminated composite plate subjected to uniform edge $\alpha = 0.2$. 
IV. CONCLUSION

The results from a study of the instability behavior of laminated composite plates subjected to uniform periodic in-plane compressive loading can be summarized as follows:

1. The onset of instability occurs later with narrow zones of instability with addition of restraint at the edges.
2. Due to static components, the instability regions tend to shift to lower frequencies, showing a destabilizing effect on the dynamic stability behavior of the composite plate.
3. Plates with large number of layers have greater dynamic stability strength.
4. The laminated composite plate becomes stiffer with a greater number of layers.

REFERENCES